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2016
MCM/ICM
Summary Sheet

Help the Earth with Water Prediction Model

In this paper, we build a water availability model and perform various simulations to study the water supply and demand of a certain region according to the International Clean water Movement (ICM).

For Task1, we generally consider the dominant role in deciding the demand and supply of water resource. We find that demand can be estimated by population, GDP and cultivated land. Water supply mainly comes from directly utilizable clean water and purified water by human, which can be forwardly classified into made-clean water, transferred water and desalinated water.

For Task 2, we specify the region to India as our target, and after analyzing the river, terrain, climate, administrative districts and population, we divide it into five main river systems. Incorporating with the natural and artificial factors including climate, terrain, precipitation, agriculture, industry, etc. perspectives to analyze the water scarcity situation in different parts of India.

For Task 3, we firstly quantify our model based on the region we choose. Then we use this model to depict the water situation in the future. For the outcome we get, we discuss its reason and detail its effect.

For Task 4, we talk about using genetic algorithm to get the optimized solution for our model. Based on this optimized solution, we give our intervention plan. We also discuss the effect of this plan in a larger context and its strengths and weaknesses.

For Task 5, we run simulations based the effect of our intervention plan and compare them with that without the intervention plan. We discuss about the differences and analyze how our intervention plan can mitigate water scarcity in the future.

Contents

1. Introduction

1.1 Problem Statement

We are facing a lot of existing challenges nowadays, and one of the most urgent problems are the scarcity of water resource. Water is already over-appropriated in many regions of the world. Water represents an essential factor for all the life inhabiting our planet, but we are now facing serious problems of water scarcity. [1] According to the United Nations, about 1.6 billion people experience lack of water. Water scarcity is among the contemporary problems of our time across the globe. Water use has been growing at twice the rate of population over the last century. Water sustains humans' agriculture and, thus, our food chain. Humans require water resources for industrial, agricultural, and residential purposes.

There are two primary kinds of water scarcity: physical scarcity and economic scarcity. Yet water is becoming scarcer globally and every indication is that it will become even more so in the future. Population growth and economic development are driving significant increases in agricultural and industrial demand for water. Agriculture accounts for more than two-thirds of global water use, including as much as 90 percent in developing countries. Decreasing availability, declining quality, and growing demand for water are creating significant challenges to provide enough fresh water to meet the needs of it.

1.2 Definition of Clean Water Supply and Demand

The renewable fresh water supply includes the surface water, underground water and artificial water supply produced by human. Surface water is a river, a lake or a freshwater wetland of natural precipitation by years and snow accumulation and natural loss to the ocean or via evaporation dies, and seepage to the ground. Underground water is also only a part of the shallow aquifer can be harvested by human. In this way, after searching data of a certain region or country, we choose India as our target place, of which most parts are heavily overloaded or moderately overloaded, according to the instructions of this task.

Clean water means that the water can be utilized by human activities, used for agriculture, residential living, and industry. Since different country has various levels of technology in water harvesting, saving and reusing, the proportion of clean fresh water varies, and this rate can range from about 30% to 70%, which indicates the ability of clean fresh water supply in a certain country or region.

2. Assumptions and Notations

2.1 Basic Assumptions

In order to have a better study on this paper, we simplify our model by the following assumptions:

1. The water supply of nature remains the same in a certain region.
2. There are no droughts, floods and strong disasters happen.
3. Water transfer line is straight, and water diversion costs per kilometer ignoring the impact of terrain.
4. Not considering the inflation of currency or bankruptcy of government.
5. Not considering increased or decreased freshwater according to greenhouse effect.
6. The water demand of a certain region can be estimated by the value of some dominant variables.
7. All parts of a river system share the similar water condition.

2.2 Notations of the Equation

Table 1. Notations

Symbol	Definition	Unit
D	The amount of water demanded by a region in total	10^9 m^3
P	Total population of a region	1000 inhabitants
G	GDP of a region	US $
L	Total area of agricultural used land of a region	1000 ha
S_i	Runoff of the *ith* place in a region	10^9 m^3
Ro_i	The amount of clean water that can be directly used by human of the *ith* place in a region	10^9 m^3
R_i	The amount of water that is made clean from those that cannot be directly used of the ith place in a region	10^9 m^3
R_{ij}	The amount of clean water that is transferred from the ith to the jth place in a region	10^9 m^3
O_i	The amount of clean water that is desalinated from ocean water of the ith place in a region	10^9 m^3
RS_i	The ith river system that have similar water situation in a region	-

3. The Water Availability Model

In order to develop a model to measure the water availability in a region, we can divide this problem into two parts, which can be noted as supply and demand.

3.1 The Demand of Water Resource

We consider that the demand of water resource roughly equals the total water withdrawal, which can be divided into three parts: Agricultural water withdrawal, Industrial water withdrawal, and Municipal water withdrawal. Cultivated Land, GDP, and Population are used to reflect each parts. The water demand D can be derived by **Equation** 1, in which P stands for population, G stands for GDP; L stands for area of cultivated land, and parameters α_i are to be determined.

$$D = \alpha_1 P + \alpha_2 G + \alpha_3 L \tag{1}$$

3.2 The Supply of Water Resource

As for water supply S_{i}, we can divide it into two parts according to their origins, Nature and Human. The Nature part Ro_i refers to water recourse that people can get with nearly no cost and it is mainly affected by the environmental condition, which remains the same during a short time period; The Human part refers to water resource that people can get with certain amount of cost. We generally take three variables into our consideration, which are River Treatment, Water Transfer and Seawater Desalination. The supply analysis can be quite complicated as environmental condition may influence the difficulty of fetching water and people's activity may in turn affect the environment. Numerically, we estimate the supply of a certain region by the following equation:

$$S_i = Ro_i + R_i + \sum_{i \neq j} R_{ji} + O_i \tag{2}$$

R_i stands for water resource through river treatment method; R_{ji} stands for water transferred from region j to region i; O_i stands for water resource through seawater desalination method.

4 Specify the Region Research

In response to the requirement of the International Clean water Movement (ICM), we pick India as our target region to analyze, and provide a model for it.

4.1 The Water Problem in India

According to the UN water scarcity map, India is now heavily and moderately exploited in most parts.

India is located in the subtropical and tropical regions, and the national average annual rainfall is of 1170 mm. With the impact of the northern Himalaya Range and the Southern Ocean, the precipitation distribution in the time and area is very uneven. Northeast and along the coastal regions of precipitation, such as Assam's average annual rainfall is over 4000 mm; in hinterland of the Indian peninsula such as Pull Jia Stan desert, precipitation is only 100 to 200 mm per year. [2] Duration of rainfall is greatly affected by monsoon climate, and up to 70% of the rainfall is concentrated in June to September.

The annual total amount of surface water resources is about 1880 km^3, which can only be effectively used 690 km^3, adding the situation of large water usage of agriculture and heavily water pollution, water resource per capita is about 2300 m^3/year, only 1/5 of the world's average. [3]

4.2 India Partition According to Water Resources

Partition principles:

1. Try to take the integrity of the administrative divisions into considerations, which is easier to collect data.
2. Each part has the same level of annual precipitation and a relatively consistent water system.
3. Maximally divided together the coastal areas, to be convenient for subsequent desalination process.

India has a lot of rivers and lakes, and there are five major river systems: the Indus River, the Ganges River, the Brahmaputra River, East Coast and West Coast rivers.

India has 34 states and unions, which can be divide into several river systems. With comprehensive considerations of geographical location, climate, economy, population, terrain and other factors, India was divided into 5 River Systems(RS):

RS1: the Indus River system;

RS2: the Ganges River system;

RS3: the Brahma-putra River system;

RS4: East Coast river system;

RS5: West Coast river system

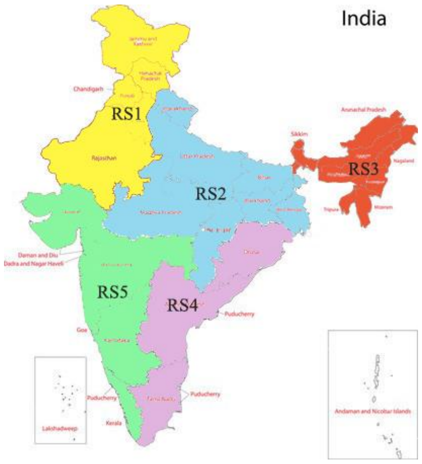

Figure 1. River System Division in India

Table 2. India Partition

		States/Union territories	Code
The Indus River	State	Rajasthan	RJ
		Haryana	HR
		Punjab	PB
		Himachal Pradesh	HP
		Jammu and Kashmir	JK

	Union	Chandigarh	CH
		National Capital Territory of Delhi	DL
The Ganges River	State	Chhattisgarh	CG
		Madhya Pradesh	MP
		Uttar Pradesh	UP
		Jharkhand	JH
		Bihar	BR
		West Bengal	WB
		Uttarakhand	UK
The Brahma-putra River	State	Sikkim	SK
		Meghalaya	ML
		Tripura	TR
		Arunachal Pradesh	AR
		Assam	AS
		Manipur	MN
		Mizoram	MZ
		Nagaland	NL
East Coast Rivers	State	Tamil Nadu	TN
		Andhra Pradesh	AP
		Telangana	TS
		Odisha	OD
	Union	Puducherry	PY
West Coast Rivers	State	Gujarat	GJ
		Maharashtra	MH
		Karnataka	KA
		Kerala	KL
		Goa	GA
	Union	Dadra and Nagar Haveli	DN
		Daman and Diu	DD

4.3 The Analysis of Water Scarcity in India

1. The ground water will be quickly dug out.

India media analysts pointed out that in the last century 70's to 80’s, the government and municipal departments are just laying the water supply, water pipelines, but the water source is not the fundamental issue of attention and resolve.

Ground water exploitation rate is 70% faster than 20 years ago. [4] Such a large amount of mining makes it difficult to restore the water level to the site, which makes it difficult for people to use deep wells and larger pumps to collect groundwater, which is likely to hit a lot of salt or contaminated groundwater.

2. The rivers are facing being drained up.

India is now developing fast both in economy, industry and urbanization. Most of the cities in India are far from the river, with the development of urbanization, making the surrounding is not much of the lake gradually dry up. Bangalore original 262 large and small water pond, now the 181 has bottomed out. [5]

3. Farming is undoubtedly one of the most important reasons for water scarcity.

India is a developing country, and the agriculture is very important to the large population and economy. In India, farmers are now using nearly 80 percent of the country’s available water, largely from groundwater wells. The original planting crops methods, the huge amount of irrigation water demand and the especially uneven rainfall makes this problem worse.

4. The water pollution is serious.

Even the cities which are nearby the rivers, they just release the polluted water without treatment into rivers. Area of water pollution is serious, a drinking water safety is threatened, long term sewage irrigation on the other hand, resulting in fill in the surface water, groundwater, soil, agriculture, animal husbandry and fishery products, such as pollution and agricultural ecological environment destruction.

The United Nations published a survey arguing the world's water resources assessment report said that the quality of life in India 122 countries ranked 120th in the world. [6] Due to inadequate investment in water resources, India has about 2000000 tons of industrial waste water directly discharged into rivers, lakes or underground, resulting in a large area of groundwater pollution. In addition, untreated domestic water directly discharged, but also one of the important causes of water pollution. At present, known as India's "sacred river Ganga already was included in the list of the world's most polluted rivers, a direct threat to the health of hundreds of millions of residents, diarrhea, hepatitis, typhoid and cholera and other diseases in India unabated. [7]

5. Climate change makes water become rarer.

(a) Climate change will decrease natural water storage capacity from glacier/snowcap melting, thus affecting RS1, RS2, and RS3 in water supply, and will harm the population that lives near these river basins.

(b) Increase water scarcity due to changes in precipitation patterns and intensity. In particular, the subtropics and mid-latitudes, where most part of India, are expected to become substantially drier, resulting in heightened water scarcity.

(c) Increase the vulnerability of ecosystems due to temperature increases, changes in precipitation patterns, frequent severe weather events, and prolonged droughts. This will further diminish the ability of natural systems in India to filter water and create buffers to flooding.

(d) Affect the capacity and reliability of water supply infrastructure due to flooding, extreme weather, and sea level rise.

To sum up, India is facing serious water scarcity, and RS1,4,5 is physical water scarcity, RS2,3 is economic water scarcity.

5 Water Situation Prediction

5.1 Model Prediction

To determine the parameters α_i in **Equation** 1, we collect the data:

Table 3. Water Withdrawal, Population, GDP, Cultivated Area of India (FAO QUASTAT)

Year	Total water withdrawal (10^9 m^3/year)	Total population (1000 inhab)	GDP (current US$)	Cultivated area (1000 ha)
1992	522.08	903750	$2.93*10^{11}$	170006
1997	577.28	990460	$4.23*10^{11}$	170101
2002	655.98	1076706	$5.24*10^{11}$	170032
2007	715.82	1159095	$1.24*10^{12}$	169202
2012	791.12	1236687	$1.83*10^{12}$	169346
2014	821.24	1267402	$2.07*10^{12}$	170029

These parameters have different units, which will make the linear coefficients vary from each other on a large scale if they are regressed directly. So we should normalize them before the regressing process.

Then by regressing these data in MATLAB, we get the numerical expression as:

$$\mathrm{D} = 1.2879P + 0.0105G + 0.2783L - 3.0894 \tag{3}$$

The F testing significance F = 0.0027<0.005, so the original assumption can be rejected, indicating that the r^2 (Goodness-of-fit coefficient) is significant. Therefore, the established binary linear regression model is effective.

We evaluate the independence of each variable by T testing using MATLAB. Results show that for each pair of variables, H=1. This means that original hypothesis assumption is rejected. Each variable is independent of the other.

In order to predict water demand for a required year, we should also find out the relationship between each variable and time. This can be realized in MATLAB using polynomial fitting.

$$\mathrm{P} = 1601 * \mathrm{year} - 3.098 * 10\hat{}6 \tag{4}$$

$$\mathrm{G} = 22.05 * \mathrm{year}^2 - 8.724 * 10^4 * \mathrm{year} + 8.63 * 10\hat{}07 \tag{5}$$

$$\mathrm{L} = 1.697 * 10^5 + 512.6 * \cos(0.2991 * \mathrm{year}) + 313.5 * \sin(0.2991 * \mathrm{year}) + 297.9 * \cos(2 * 0.2991 * \mathrm{year}) - 771 * \sin(2 * 0.2991 * \mathrm{year}) \tag{6}$$

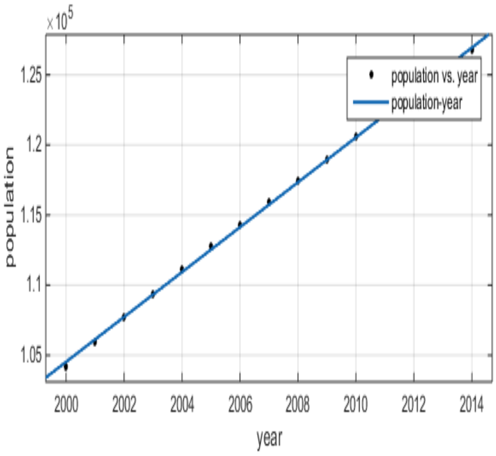

Figure 2. Population and Time

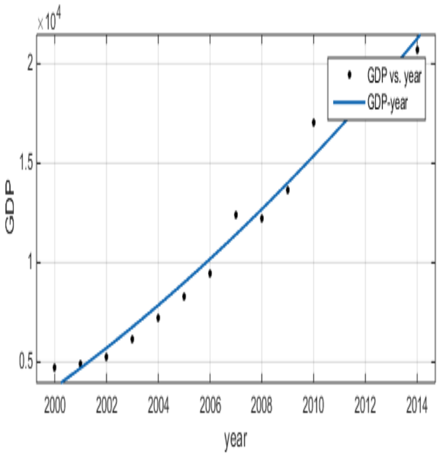

Figure 3. GDP and Time

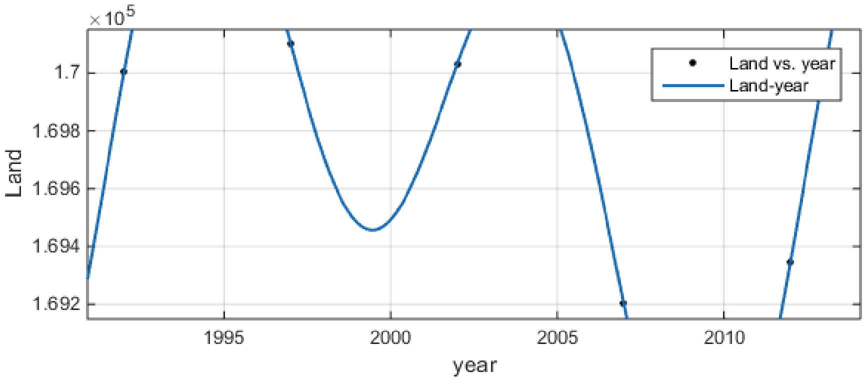

Figure 4. Cultivated Land and Time

Adjusted R-square is 0.9995, 0.9708,and 1, which means they are well fitted.

So far, we are able to predict water demand for a selected year, regarding year as an independent variable. For example, from **Figure 5**, water demand in year 2030 will be 973.1511(*10^9 m^3).

Next, to evaluate water situation in the future, we still have to predict the water supply. Data show that the water supply does not change significantly with time. This is understandable, because it won't change significantly unless the environmental condition is altered significantly. We consider the water supply as a constant equaling to 700(*10^9 m^3), according to **Table 4**.

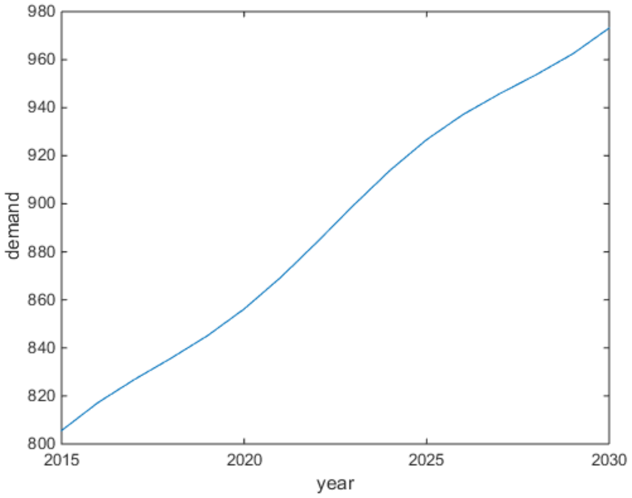

Figure 5. The Relationship between Demand and Year

Table 4. Annual Runoff & Utilizable Water Resource of Major River System in India (Baidu Baike) (×$10^9 m^3$)

River System	Annual Runoff	Utilizable Water Resource
The Indus River	77	46
The Ganges River	510	250
The Brahmaputra River	540	24
East Coast	348	281
West Coast	305	99
Total	1780	700

5.2 The Impact of Water Situation

From the outcome of the model, we can figure out that water supply can definitely not meet its demand, which is consistent to the survey and data of the World Bank, and it estimates that India will have exhausted available water supplies by 2050.

This model is very effective for people to have a preview of the water situation in 15 years, so they can take actions right before the final consequence falling:

1) Their will experience lack of drinkable water.

If they take no measures to improve this situation, the water scarcity will be much more serious than nowadays, and they will finally have little water to drink. The water quantity may be large, but not plenty clean water due to water pollution, climate change, population accumulation and agriculture irrigation, etc. The lack of clean water may lead to conflicts for water, influencing the peace and order of this region.

2) The health problem with be critical.

Living in an environment without enough clean water will harm the health of citizens of this region. As important as water is to human, the life of those citizens will be hard, and they will even pay much higher price for small amount of water from some illegal sellers, which will increase also their burden.

3) They will prevent water pollution and clean the polluted rivers.

   Since the water pollution is serious at present, and if this trend continues, the rivers will be no longer suitable to be utilized by human. The clean fresh water is important to residential life, agriculture and industry. In 15 years, the wastewater treatment techniques will be more advanced, and the government will invest more money into ameliorating rivers' water quality.

4) It is clear that human must actively search for clean water.

   They can detect the surrounding areas for new aquifer, or they can desalinate sea water in order to supply the demand of clean fresh water, but it will cost much more than they pay for water today, and will make their life less convenient.

6 Measures of India

6.1 Separate the Demand and the Supply

Since five water-system regions in India differ greatly in water scarcity, we put forward a detailed intervention plan among those five regions, taking all the drivers of water scarcity into account. Therefore, we also separate the demand and the supply of water resource to five parts corresponding to five water regions.

We use d_{i} to represent the proportion of water resource demand in five water regions. From **Equation 3** we can reach the conclusion that the demand of water resource depend largely on the amount of cultivated land. In fact, agricultural water withdrawal accounts for more than 90% water usage in developing countries, especially in India. We choose the agricultural water demand to be approximately equal to water demand in each region. Furthermore, the agricultural water demand can be estimated by area of cultivated land. According to survey data, the proportions of Indian farm land in plain regions and plateau/mountainous regions is 93% and 7%, respectively. After calculating the area of different terrains in various districts, we got the area of cultivated land.

$$d_i = \frac{\text{demand of region } i}{\text{demand of India}} \approx \frac{\text{cultivated land of region } i}{\text{cultivated land of India}} \tag{7}$$

We separate the supply of water resource according to the third row of **Table 4**. One river system that has more utilizable water resource leads to the fact that there’s more supply of water resource in that region.

6.2 Decrease the Demand

The demand of water comes from three main parts: agriculture, industry and residential life. To decrease the water demand, we should find ways in the above field:

- Agriculture: It accounts for most of the country's water demand. Using new irrigation techniques, for example, will decline the waste of water and make planting more efficient.
- Industry: It brings about the most severe water pollution, such as heavy metal pollution and organic pollution, affecting the water quality. The demand of industry cannot be easily decreased, but the treatment of polluted industrial water is essential. With the wastewater treated, the air quality can maintain, so as to reserve the amount of clean water, which is a kind of water demand reduction.
- Residential life: The citizens can save water in daily life, turning off the tap after use, strengthening their opinions of reduce water usage. The wastewater flushed to the sewage pipes also needs to be processed before being released to environment.

6.3 Increase the Supply

As discussed in **Section 3.2**, the supply of water resource is divided into two part: natural and artificial. For nature part Ro_i, there are different environmental, economic, and social factors.

Take the environmental factors for example, the land dissertation, the random and uneven precipitation in different districts, and the climate change may affect the water supply in various ways. The climate change will likely to melt more ice from mountain peaks, which will increase the supply of water to inner rivers.

The economic factors are another important element to increase water supply. With the development of economy, the government can invest more money to build desalination factories to provide more fresh water, and the companies can improve their water harvesting techniques to obtain more water.

When we do the computer simulation using MATLAB, we regard Ro_i as a constant, which is 700($*10^9 m^3$) according to **Table 4**, and will not introduce any cost.

For human part, we consider three most widely-used methods: river treatment, water transfer, and seawater desalination.

6.3.1 River Treatment Method

We use the ratio of utilizable water resource Ro_i and annual runoff S_i from **Table 4** to present the water quality. Through river treatment method, the water quality will increase, thus more utilizable water resource will be available. It is reasonable that it cost more to improve the water quality of a heavily polluted river. We define the unit cost factor a_i for river treatment method with water quality Ro_i/S_i:

$$a_i = 6 * \left(0.5 - \frac{Ro_i}{S_i}\right)^3 + 0.75 \tag{8}$$

When water quality is very close to 1, it indicates that the river is quite clean and there is no need to implement river treatment, so the unit cost factor a_i should be 0. When water quality is close to 0, the unit cost factor $a_i > 1$, because the unit cost of treating a heavily polluted river would be larger than the average unit cost.

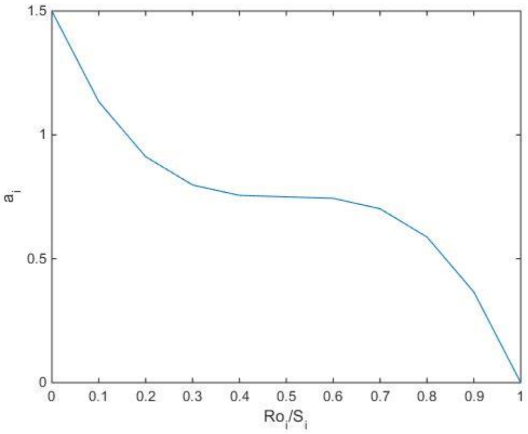

Figure 6. The unit cost factor a_i decreases while the water quality Ro_i/S_i increases, and the unit cost factor a_i would reach its minimum 0 when the water quality Ro_i/S_i reaches its maximum 1.

Depending on current technology, we set the unit cost of river treatment method *A* to 50 (Indian Rupee/m^3) for India. We use R_i to represent the amount of water we get in each region through river treatment method.

6.3.2 Water Transfer Method

As a long distance leads to a high cost, we define unit cost factor b_i for water transfer method:

$$b_i = \frac{\text{real distance}}{\text{base distance}} \tag{9}$$

The water transfer cost of south-to-north water diversion project center line is 2.6(yuan/m^3) [8]. We use it to estimate the unit cost *B* = 27(Indian Rupee/m^3). Thus, the corresponding base distance is 1432(km).

We use R_{ij} to represent the amount of water transferred form region i to region j. Then there is the relation $R_{ij} = -R_{ji}$.

6.3.3 Seawater Desalination Method

If a region is near the ocean, desalinators can be built to improve water scarcity. Once a plant is built, the unit cost should remain the same, regardless of its location. So the unit cost factor c_i for seawater desalination method is a constant 1. We estimate the unit cost C = 34(Indian Rupee/m^3). We use O_i to represent the amount of water we get in each region through seawater desalination method.

We apply Genetic Algorithm (GA for short) to get the optimum solution, that we can reach balance between supply and demand with smallest cost. The objective function is defined as:

$$\text{Total Cost} = A * \sum_i a_i R_i + B \sum_{i \le j} b_i |R_{ij}| + C \sum_i c_i O_i \tag{10}$$

There are several constraints that can be expressed as **Equation 11** and **Equation 12**. After the intervention plan is carried out, the supply of water resource should be no less than the demand of water resource in each region. The amount of water we get R_i through river treatment method could not break its top limit.

$$R_{oi} + R_i + \sum_{i \neq j} R_{ji} + O_i \ge d_i D \tag{11}$$

$$0 \le R_i \le S_i - R_{oi} \tag{12}$$

6.4 Solution and Analysis

After the first time of optimization, we find that $R_{13}, R_{14}, R_{23}, R_{34}, R_{35} \approx 0$, which means it is only necessary to build five channels for water transfer, linking region 1(yellow area) and region 2(blue area), region 1(yellow area) and region 5(green area), region 2(blue area) and region 4(purple area), region 2(blue area) and region 5(green area), region 4(purple area) and region 5(green area).

Add these constraints; we get our intervention plan as displayed in **Figure 7**. The Indus River Region (yellow area) needs to get 30.7(*10^{9}m^3) water resource through river treatment method. The West Coast Region (green area) needs to get 30.9(*10^{9}m^3) water resource through river treatment method, 30.9(*10^{9}m^3) water resource through seawater desalination method and transfer 32.6(*10^{9}m^3) water resource to the Indus River Region. The East Coast Region (purple area) needs to get 4.3(*10^{9}m^3) water resource through seawater desalination method and transfer 16.2(*10^{9}m^3) water resource to the Ganges River Region, 30.9(*10^{9}m^3) water resource to the West Coast Region. The Ganges River Region needs to get 16.2(*10^{9}m^3) water resource through river treatment method and transfer 48.3(*10^{9}m^3) water resource to the Indus River Region, 14.7(*10^{9}m^3) water resource to the West Coast Region. The Brahmaputra River Region needs to get 18.4(*10^{9}m^3) water resource through river treatment method and 19.4(*10^{9}m^3) water resource through seawater desalination method.

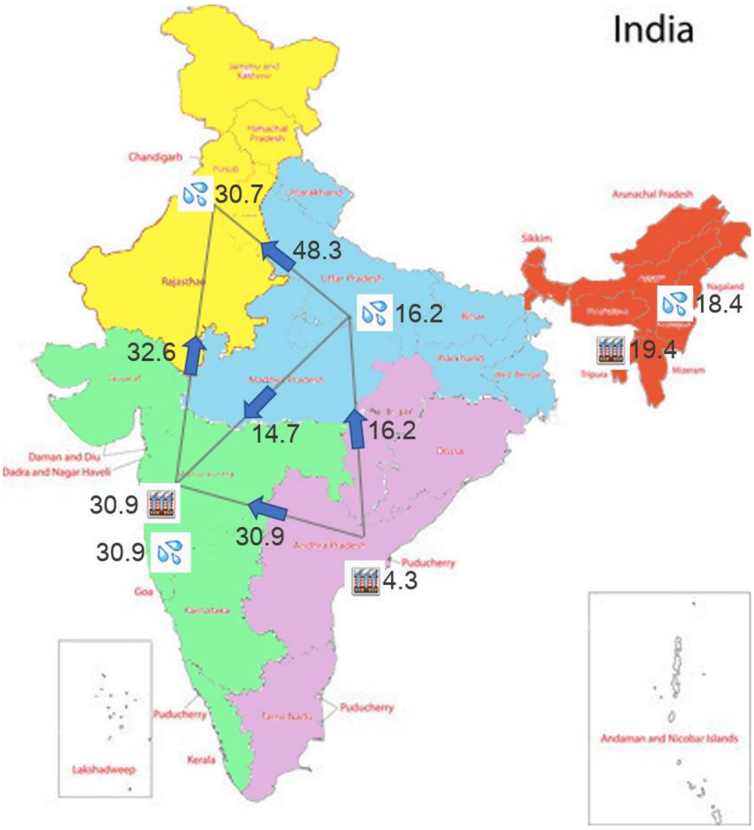

Figure 7. The Intervention Plan

As soon as the construction of those desalinators and water channels complete, all five regions in India will have no water scarcity that the supply of water resource can meet the demand.

In our intervention plan, the river treatment method can improve the water quality, which generally would not change in the future. So it can mitigate water scarcity in the future without any further expense. But in certain region, such as the Indus River Region, it'll be more complicated to try to improve water quality, because not the entire river is within India; the cooperation with neighboring countries would be inevitable. Besides, water channels may change the water ecosystem in India or in Southeast Asia. Desalinators may consume fossil fuels to get water resource, which might increase carbon emission, resulting worse greenhouse effect.

7 Model Evaluation

7.1 Evaluation Using AHP Model

In order to mitigate water scarcity, water transfer, river treatment, and seawater desalination are considered as three main methods to increase the amount of water supply. Taking four criteria into account, our AHP model for evaluation is illustrated in **Figure 8**.

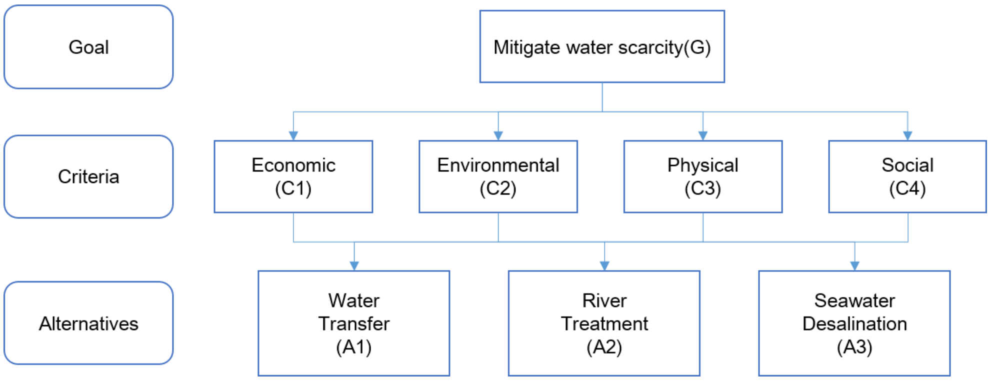

Figure 8. The AHP model for mitigating water scarcity.

To get each weight vector, we set the pairwise comparison matrices as follows:

$$\begin{array}{ccccc} G & C_1 & C_2 & C_3 & C_4 \\ C_1 & 1 & 9/5 & 9/7 & 7/5 \\ C_2 & 5/9 & 1 & 3/4 & 2/3 \\ C_3 & 7/9 & 4/3 & 1 & 9/8 \\ C_4 & 5/7 & 3/2 & 8/9 & 1 \end{array} \qquad \begin{array}{cccc} C_1 & A_1 & A_2 & A_3 \\ A_1 & 1 & 7/2 & 7 \\ A_2 & 2/7 & 1 & 1 \\ A_3 & 5/7 & 3 & 1 \end{array}$$

$$\begin{array}{cccc} C_2 & A_1 & A_2 & A_3 \\ A_1 & 1 & 1/2 & 8/9 \\ A_2 & 2 & 1 & 2 \\ A_3 & 9/8 & 1/2 & 1 \end{array} \qquad \begin{array}{cccc} C_3 & A_1 & A_2 & A_3 \\ A_1 & 1 & 3/5 & 8/9 \\ A_2 & 5/3 & 1 & 7/9 \\ A_3 & 9/8 & 9/7 & 1 \end{array} \qquad \begin{array}{cccc} C_4 & A_1 & A_2 & A_3 \\ A_1 & 1 & 3 & 2 \\ A_2 & 1/3 & 1 & 3/7 \\ A_3 & 1/2 & 7/3 & 1 \end{array}$$

Then by using MATLAB, we can respectively get the weight vector, the Consistency Index $CI = (\lambda_{\max} - n)/(n-1)$ and the Consistency Ratio $CR = CI / RI$. The constant $RI = 0.58$ when $n = 3$ and $RI = 0.90$ when $n = 4$.

Table 5. Weight vector, *CI*, and *CR*.

	G
C_1	0.3273
C_2	0.1774
C_3	0.2538
C_4	0.2414
CI	0.0017
CR	0.0019

	C_1	C_2	C_3	C_4
A_1	0.4931	0.2403	0.2674	0.5356
A_2	0.1326	0.4998	0.3596	0.1541
A_3	0.3743	0.2599	0.3730	0.3103
CI	0.0018	0.0008	0.0232	0.0109
CR	0.0032	0.0013	0.0399	0.0187

Then the result can be obtained by **Equation 13**:

$$W_i = \sum_{j=1}^{4} W_{C_j, A_i} * W_{G, C_j} \quad i = 1, 2, 3 \tag{13}$$

The weights for each alternatives are $W_1 = 0.4012$, $W_2 = 0.2605$, $W_3 = 0.3743$, which means the water transfer method has the greatest importance while the river treatment method has the smallest importance. This is consistent with our intervention plan.

Consistency Check:

We test the consistency of our AHP model in two aspects: *CI* and *CR*. *CI* should be close to zero; *CR* should be less than 0.1. The *CR* for the model can be calculated by **Equation 14**, in which the parameters α_i (i=1,2,3,4) are the elements of the weight vector.

$$CR = \frac{\alpha_1 CI_1 + \alpha_2 CI_2 + \alpha_3 CI_3 + \alpha_4 CI_4}{\alpha_1 RI_1 + \alpha_2 RI_2 + \alpha_3 RI_3 + \alpha_4 RI_4} \tag{14}$$

According to the **Table 5**, all *CI* are quite close to zero, and $CR = 0.0159 < 0.1$, so the consistency is perfectly acceptable.

7.2 Sensitivity Analysis

We choose the unit cost factor a_i for river treatment method to be checked. In terms of the relation between a_i and the water quality Ro_i/S_i, besides polynomial function, we also use tangent function displayed as **Equation 15** and hyperbolic sine function displayed as **Equation 16** for mapping to test sensitivity.

$$a_i = 0.6495 * \tan\left(0.9 * \pi * \left(0.374 - 0.5592 * \frac{R_i}{S_i}\right)\right) + 0.3645 \tag{15}$$

$$a_i = 0.7055 * \left(e^{0.7 - \frac{R_i}{S_i}} - e^{-\left(0.7 - \frac{R_i}{S_i}\right)}\right) + 0.4296 \tag{16}$$

It turns out that the value of a_i dose not cause great changes to our results.

Table 6. Sensitivity Analysis of a_i

Mapping Method	Polynomial	Tangent	Hyperbolic Sine
Total Cost	6994.1	7160.7	6423.8

8 Water Situation Prediction after the Intervention Plan

The intervention plan we put forward is assumed to be carried out in 2015. This plan will alter the environmental condition mainly by increasing the utilizable water resource. We measure water scarcity by calculating Water Scarcity Indicator (*WSI* for short) of a region, which is the ratio of demand and supply. *WSI* = 1 means that the nature currently cannot satisfy the demand of water resource. We run simulations and calculate *WSI* for India and its five regions from the year 2015 to 2030.

The red line shows *WSI* without the effect of our intervention plan while the blue line shows the effect. It can be seen that the situation for each region is improved more or less with the fact that blue lines are all below their corresponding red lines.

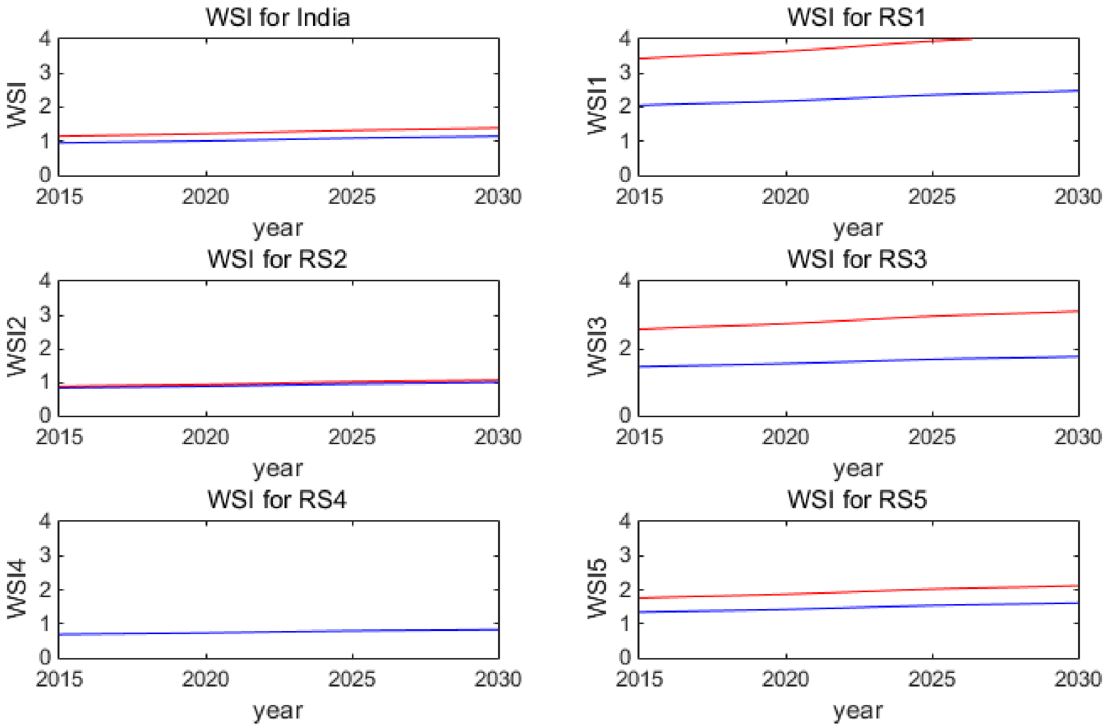

Figure 9. *WSI* Prediction with/without the Intervention Plan

Generally speaking, our plan solves the water scarcity problem in 2015 from the country’s view. But seen from each region of India, this problem may still exist but greatly relieved. Considering that our plan pursues the least cost, water scarcity problem will soon come back if nothing has been done after our intervention plan. From the **Figure 9** we can estimate that the water scarcity will occur again at about the year of 2019. So, it is highly recommended that this plan should be taken every time there is going to be a water scarcity problem. This may be considered as the best choice because problems will be solved at the lowest cost.

9 Strengths and Weaknesses

9.1 Strengths

- Comprehensiveness

Our model looks deep into dominant roles that play important parts in this problem and derive relatively accurate methods to quantify them, so that they can easily be used in real life.

Our model does not let go of some minor factors that also play a part in this problem. We carefully study and estimate them and put them into our model.

Our model is based on scientific methods and assumptions. Therefore it is reliable and understandable. It is also easy to be improved or modified.

Our model considers the dynamic natural of the reality, it is quite open for new situations.

- Robustness

Sensitivity analysis proves that a slight change in parameters will not cause significant changes in our results. Our model is fairly robust.

- Justifiability

Our model will be consistent with our common feelings. For parts that are difficult or actually impossible to judge we figure out ways to make them real.

9.2 Weaknesses

- Limited by data we get, some parts could have been made even more accurate.
- Our model may not give exactly the same results if the condition is unchanged, which may be annoying under some circumstances.

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